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Corrigendum to "Weighted L^p estimates for the elliptic Schrödinger operator" [*Electron. J. Qual. Theory Differ. Equ.* 2014, No. 33, 1–13]

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Abstract. An error in Lemma 1.5 in "Weighted L^p estimates for the elliptic Schrödinger operator" [*Electron. J. Qual. Theory Differ. Equ.* 2014, No. 33, 1–13] is corrected.

Keywords: weighted, regularity, L^p estimates, elliptic, Schrödinger operator.

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The author regrets to inform that Lemma 1.5 in [1] is not fully correct. Since the main results in [1] eventually rely on this lemma, we shall give the following new lemma instead of the wrong Lemma 1.5 in [1] and then put it behind Lemma 1.9 in [1].

Lemma 1.9. *Assume that $w \in A_q$ for some $q > 1$.*

(1) There exists a positive constant $q_1 \in (1, q)$ such that

$$w \in A_{q_1}.$$

(2) Let $q_2 = \frac{q}{q_1} \in (1, q)$. Then we have

$$L^q_w(B_r) \subset L^{q_2}(B_r).$$

Proof. Since $w \in A_q$, from Definition 1.3 in [1] we have

$$\begin{aligned}
&\left(\fint_{B_r} w(x)\,dx\right)\left(\fint_{B_r} w(x)^{\frac{-1}{q-1}}\,dx\right)^{q-1} \\
&\quad = \left(\fint_{B_r} \left(w(x)^{\frac{-1}{q-1}}\right)^{1-q} dx\right)\left(\fint_{B_r} w(x)^{\frac{-1}{q-1}}\,dx\right)^{q-1} \\
&\quad = \left[\left(\fint_{B_r} \left(w(x)^{\frac{-1}{q-1}}\right)^{-\frac{1}{\frac{q}{q-1}-1}} dx\right)^{\frac{q}{q-1}-1}\left(\fint_{B_r} w(x)^{\frac{-1}{q-1}}\,dx\right)\right]^{q-1} \leq C \qquad (1.1)
\end{aligned}$$

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for any balls B_r in $\mathbb{R}^n$, which implies that $w(x)^{\frac{-1}{q-1}} \in A_{\frac{q}{q-1}}$. Therefore, from Lemma 1.8 in [1] we have

$$\left(\fint_{B_r} w(x)^{-\frac{1+\epsilon_0'}{q-1}}\,dx\right)^{\frac{1}{1+\epsilon_0'}} \le C\fint_{B_r} w(x)^{\frac{-1}{q-1}}\,dx \tag{1.2}$$

for some $\epsilon_0' \in (0,1)$. Let

$$q_1 = 1 + \frac{q-1}{1+\epsilon_0'} \in (1,q).$$

Then from (1.2) and the fact that $w \in A_q$ we find that

$$\begin{aligned} &\left(\fint_{B_r} w(x)\,dx\right)\left(\fint_{B_r} w(x)^{\frac{-1}{q_1-1}}dx\right)^{q_1-1} \\ &= \left(\fint_{B_r} w(x)\,dx\right)\left(\fint_{B_r} w(x)^{-\frac{1+\epsilon_0'}{q-1}}\,dx\right)^{\frac{q-1}{1+\epsilon_0'}} \\ &\le C\left(\fint_{B_r} w(x)\,dx\right)\left(\fint_{B_r} w(x)^{-\frac{1}{q-1}}dx\right)^{q-1} \le C, \end{aligned} \tag{1.3}$$

which implies that $w \in A_{q_1}$. Furthermore, if $f \in L^q_w(B_r)$, then from Hölder's inequality and (1.3) we have

$$\begin{aligned} \int_{B_r} |f|^{\frac{q}{q_1}}\,dx &= \int_{B_r} |f|^{\frac{q}{q_1}}\, w(x)^{\frac{1}{q_1}} w(x)^{-\frac{1}{q_1}}\,dx \\ &\le \left(\int_{B_r} |f|^q\, w(x)\,dx\right)^{\frac{1}{q_1}} \left(\int_{B_r} w(x)^{-\frac{1}{q_1-1}}dx\right)^{1-\frac{1}{q_1}} \\ &\le C\left(\int_{B_r} |f|^q\, w(x)\,dx\right)^{\frac{1}{q_1}} \left(\frac{|B_r|}{w(B_r)}\right)^{\frac{1}{q_1}} \le C, \end{aligned}$$

since $w \in L^1_{\text{loc}}(\mathbb{R}^n)$ and $w > 0$ almost everywhere. This finishes our proof by choosing $q_2 = \frac{q}{q_1} \in (1,q)$. □

We shall add the following sentences in front of "Next, we shall prove the following important result" in [1], page 5, line -6:

"Assume that $w \in A_p$. Then from Lemma 1.9 (1) (see above) we find that

$$w \in A_{p_1} \quad \text{for some} \quad p_1 \in (1,p).\text{"} \tag{1.4}$$

Moreover, we shall change "Assume that $1 < q < p$" in Lemma 2.2 of [1] to "Assume that $w \in A_p$" and add the sentence "where $q = \frac{p}{p_1} \in (1,p)$ and p_1 is defined in (1.4)," behind (2.2) in [1]. Furthermore, we shall change "Assume that $1 < q < p$ and $w \in A_p$" in Corollary 2.3 of [1] to "Assume that $w \in A_p$" and add the sentence "where $q = \frac{p}{p_1} \in (1,p)$ and p_1 is defined in (1.4)," behind (2.8) in [1].

The author would like to apologize for any inconvenience caused.

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References

[1] F. Yao, Weighted L^p estimates for the elliptic Schrödinger operator, *Electron. J. Qual. Theory Differ. Equ.* **2014**, No. 33, 1–13. MR3250024; url